Stanford Math Directed Reading Program Colloquium
Spring 2023

June 8, 6:30pm–8:30pm
Sloan Mathematics Center, room 384H (fourth floor)
Dinner available at 5:30pm

An Introduction to Models of First-Order Theories and Categoricity

Álvaro Díaz Ramos
Mentor: Spencer Dembner

Model theory is the branch of mathematical logic concerned with the study of mathematical structures through the lens of their first-order properties. A foundational question that naturally arises is the characterization of models with cardinality κ of a given theory up to isomorphism (categoricity). In this talk, I will provide an introduction to model theory, concluding with a discussion on a key result by Michael D. Morley, which demonstrates that there is only one notion of uncountable categoricity.

Understanding Matrix Multiplication

Eban Ebssa
Mentor: Talia Blum

Sometimes, people (including myself before DRP) see matrix multiplication as a matter of arithmetic and memorization. In this talk, we will try to understand and visualize matrix multiplication. Then, we will apply our understanding to the first part of the proof of the Fundamental Theorem of Linear Algebra, a matrix decomposition formula relevant to data science, signal processing, and machine learning.

Making Physics Rigorous: Introduction to TQFTs

Eugenie Shi and Iris Zhou
Mentor: Shintaro Fushida-Hardy

Abstract: In quantum mechanics, physicists often use a path integral formulation to associate observable states to measurable quantities. Unfortunately, it turns out that these path integrals don't mathematically make sense, so we as mathematicians are going to help the physicists make their ideas mathematically rigorous. We introduce the idea of a cobordism as a model for observable states, and apply a categorical approach to create a "mapping" (formally, a functor) between cobordisms and vector spaces and linear maps. The vector spaces and linear maps represent measurable quantities, and this "mapping" (functor) is called a TQFT. Finally, we give several low-dimensional examples of TQFTs.

Class groups and quadratic forms

Ruosong Gao
Mentor: Yunkun Zhou

Quadratic forms, or expressions of the form $ax^2 + bxy + cy^2$ with integers a, b, c, play an important role in generalizing well-known results regarding primes that can be represented as a sum of two squares. However, it turns out that there is a surprising connection between these forms and the structure of their so-called 'composition', and classes of ideals in imaginary quadratic fields. I'll be presenting on what this connection is, and what its significance is.

Percolation, Phase Transitions, and Global Connectivity in Lattices

Benjamin Yan
Mentor: Christian D. Serio

When a meme is posted, what journey does it take through vast Internet networks, and at what critical point does it gain virality? In a different setting, when liquid is poured on the surface of some permeable solid, how can we predict if the liquid will pass through to the other side? In this talk, we explore geometric and probabilistic models of percolation. We consider a lattice model where each edge appears independently with chance p, and the probability of an infinite cluster. This is a connected embroidering of edges that covers infinitely many vertices. The critical p where such a cluster can exist with positive probability is an open question in many high-dimensional lattices. We aim to sketch this fascinating, multivalent problem, which has manifold implications for physics, biochemistry, and network science.

Stochastic Processes: A Random Walk Through Time

Rayhan Razzak
Mentor: Jared Marx-Kuo

During my time in the directed reading program, I was introduced to the field of stochastic processes, specifically Markov chains. One of my keen interests is random walks. In my presentation, I will discuss simple random walks in two dimensions (graphs) and explore various methods to determine the invariant probability to understand how random walks act in large-n time as well as explore techniques to answer questions related to expectations: expected number of steps to reach a given state when starting at another state and the difference between recurrent classes and transient classes.

Journey through Cardinality

Judy Liu and Nika Zahedi
Mentor: Romain Jacques Higham Speciel

We are going to talk about the cardinality of naturals vs rationals vs reals. In the end of our talk, we want to mention Gödel's incompleteness theorem, and trying to find a set between naturals and reals.

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