

# Stanford Math Directed Reading Program Colloquium

## Spring 2018

Thursday, June 12th, 6:30pm–8:30pm  
Sloan Mathematics Center, fourth floor  
Dinner available at 6:15pm

### Session A, room 384-H

#### Riemannian Manifolds

Alec Lau

Mentor: Scott Zhang

Here I will present some basic ideas of Riemannian Geometry and Differential Geometry. This will include the mathematical machinery of a few geometric concepts: fiber bundles, connections, and metrics. Then I will present how these are used to generalize notions of derivatives, length, and distance. Finally, I will present the Gauss-Bonnet Theorem, which establishes an interesting connection from a 2-manifold's geometry and its topology.

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#### Additive Number Theory and the Hardy-Littlewood Circle Method

Ali Malik

Mentor: Sarah Peluse

The Circle method is a beautiful technique for tackling problems in additive number theory. First introduced by Hardy and Ramanujan while investigating the asymptotics of the partition function, it has seen wide use in providing analytic bounds on various Diophantine equations. In this presentation, we introduce the Circle method and apply it to Waring's famous problem on the feasibility of writing all natural numbers as a sum of at most  $s$  perfect  $k$ -th powers.

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#### The Mathematics of the Magnetic Monopole

Sean Afshar

Mentor: Jesse Madnick

It is often the case that investigations in mathematics and physics lead to the same troves of ideas and discoveries. One such example is the magnetic monopole. I will go over the notions that Dirac was experimenting with and explain how they laid the foundation for gauge theory. Then I will go over the basics of mathematical gauge theory and tie them back to the magnetic monopole. Finally I will quickly go over the interesting physical phenomena that are implied by the possible existence of a magnetic monopole.

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# The Hopf Fibration

Matthew Stevens

Mentor: Jesse Madnick

The Hopf Fibration is a fascinating principle bundle arising from a surjective map from  $S^3$  to  $S^2$  discovered by Heinz Hopf in 1931. In this talk, we will construct the Hopf Fibration and discuss several ways of thinking about its structure. Further, we will explore a few of its beautiful geometric properties, as well as its surprising connection to the physics presented in Sean's talk.

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# A Classical Proof of Quadratic Reciprocity

Nikolas Castro

Mentor: Jesse Silliman

Quadratic reciprocity defines a relationship between the existence of solutions to the equation  $x^2 \equiv p \pmod{q}$  and existence of solutions to  $x^2 \equiv q \pmod{p}$ . There are many classical proofs of this, and in this talk I will introduce one not too dissimilar from the modern proof (which uses algebraic number theory).

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