# Stanford Math Directed Reading Program Colloquium Spring 2018

Thursday, June 7th, 6:30pm—8:30pm Sloan Mathematics Center, room 384-I (fourth floor) Dinner available at 6:15pm

### A dynamical systems perspective on the Oppenheim Conjecture: Motivations, Intuitions, and Theorems.

Raul Girbal Mentor: Cole Graham

Under the lens of Dynamical Systems, we will look at the Oppenheim Conjecture, a famous conjecture in Diophantine Approximation. Proved by Margulis in 1987 using Ergodic Theory and the theory of semi-simple Lie groups as the backbone of the proof, the conjecture is a wonderful example of the intersection of different areas of Mathematics. In this talk, we will briefly cover some necessary motivations and intuitions, the main results and some sketch proofs, and briefly discuss similar open problems.

### A short introduction to ergodic theory and Gelfand's question

Yelena Mandelshtam Mentor: Joey Zou

Ergodic theory uses tools of measure theory to study the long-term behavior of abstract dynamical systems. In my talk I will provide a brief introduction to this subject, defining strong mixing, weak mixing, and ergodicity. I will then present the Ergodic Theorem and show how it can be used to answer Gelfand's question regarding the leading digits of powers of 2.

## Algebraization of propositional and first order logic

Ben Heller Mentor: Joj Helfer

This talk will illustrate how logical systems can be studied using algebra, particularly the analogy between studying propositional logic using Boolean algebras and studying first order logic using logical categories. I will begin by introducing propositional logic, models, and theories, and showing how these ideas can be expressed in the language of Boolean algebras. Then I will state the Stone Representation theorem and the Completeness theorem, and prove their equivalence. Switching to first order logic, I will do the same things but with logical categories instead of Boolean algebras to demonstrate the analogy.

#### **Ergodic Analysis of Continued Fractions**

Adam Jaffe Mentor: Jimmy He

Ergodic theory is a very general study of asymptotics of dynamical systems, and, as such, it has many connections with different parts of mathematics and physics. In this talk, I give a brief overview of the classic results of the theory and use these to study the asymptotic properties of the continued fraction expansion of a real number.

#### **Domination in Tournaments**

Thao Nguyen Mentor: Xiaoyu He

In a round-robin tournament, each player plays against every other player in a single match, resulting in either a win or a loss. For a set of players from the tournament, domination happens if there exists another player who beats them all. In this talk, I will use the probabilistic method to prove the bounds on the minimum number of players a tournament needs, so that every set of k players from the tournament is dominated.

#### The Open Mapping Theorem for Analytic Functions

Eric Kim Mentor: Felipe Hernandez

It is well-known that in complex analysis, the winding number (or index) of a closed rectifiable curve  $\gamma:[0,1]\to\mathbb{C}$  around a point  $a\not\in\{\gamma\}$  can be defined by the integral  $\frac{1}{2\pi i}\int_{\gamma}\frac{dz}{z-a}$ . Using Cauchy's theorem, one can unveil incredible results regarding the multiplicity of solutions to the equation  $f(z)=\zeta$  where z and  $\zeta$  are restricted to small discs. A consequence is the open mapping theorem: that any non-constant analytic function maps open sets to open sets. Moreover, this implies that all bijective analytic functions have an analytic inverse.

# Applying Floer Homology

Zoe Himwich Mentor: Cédric De Groote

I will state the Arnold conjecture in symplectic topology and explain how Floer homology can be used to give an answer.

This event was financially supported by the Vice Provost for Graduate Education through the Student Projects for Intellectual Community Enhancement (SPICE) and Diversity and Inclusion Innovation Funds (DIF) programs. The organizers would like to thank Gretchen Lantz, Rose Stauder, and Elizabeth Kay for their administrative support of the DRP.