

# Stanford Math Directed Reading Program Colloquium

## Winter 2018, Session II

Wednesday, April 4, 2018, 6:30pm–8:30pm  
Sloan Mathematics Center, room 384-H (fourth floor)  
Dinner available at 6:15pm

### Introduction to Partial Differential Equations

Jeffrey Brown

Mentor: Mark Perlman

Partial differential equations (PDE) are equations with one or more partial derivatives. PDE exist as a natural means of describing various phenomena, as dynamical dependencies on a number of variables often accurately describe the complexity of the real world. It is due to this inherent complexity, however, that one finds solving such equations often impossible or arbitrarily difficult in general cases. Thus, one is compelled to develop methods of simplifying such equations into tractable equations without a loss of information. One such method, the method of characteristics, is used to address non-linear first-order PDE. By simplifying these equations into a system of ordinary differential equations and using initial conditions given by the boundary of a specified domain, one can extract the behavior of the PDE along characteristic lines and identify a comprehensive solution to it. The details of this procedure, along with some of its utility, are the subject of this talk.

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### Classifying Conics

Ariella Lee

Mentor: Laurent Côté

First I will talk about the relationship between the real projective plane and homogenisation of polynomials. Then I will show how homogenising the polynomials that give us the conic sections in  $\mathbb{R}^2$  gives us homogeneous polynomials in 3 variables of degree 2. Finally, I will show that these polynomials correspond to 3-by-3 symmetric matrices.

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### How to Add Points

Nolan Miranda

Mentor: Benjamin Fayyazuddin Ljungberg

Elliptic curves have fascinated mathematicians for years, and their applications in fields such as number theory and cryptography have led to some incredible discoveries. Finding rational points on these curves is an important step to understanding their structures, and in this presentation I will discuss the composition group law on rational points on these curves. This gives a way to derive rational points on a curve given other rational points, which I will demonstrate and then show some applications to higher-level elliptic curve concepts.

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## **Kuratowski's Closure-Complement Problem**

Milan Mosse

Mentor: Inbar Klang

In 1922, Kuratowski showed in his doctoral thesis that starting with a subset of a topological space, one can form no more than fourteen distinct sets by applying the operations of closure and complementation successively. I will give an algebraic proof of this result.

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## **Proof of The Fundamental Theorem of Algebra Using Differential Topology**

Jack Ryan

Mentor: Jens Reinhold

The Fundamental Theorem of Algebra has several proofs. I will be presenting one which utilizes notions from differential topology. To do this, I will define the notion of degree of a mapping and give examples of mappings for which degree is defined. Then, I will use a result in differential topology regarding degree and homotopy to prove the Fundamental Theorem of Algebra. The proof assumes that the Fundamental Theorem is false, then makes use of certain homotopies of maps that would exist if the theorem were false to arrive at a contradiction.

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## **Morse Homology**

Zoe Himwich

Mentor: Cédric De Groot

This talk will review of the construction of Morse homology and provide an explanation of why the differential squares to zero. In addition, the talk will work out a few examples and explain how this construction arises, as well as how and why it is used.

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