Stanford Math Directed Reading Program Colloquium
Winter 2018, Session I

Monday, April 2, 2018, 6:30pm–8:30pm
Sloan Mathematics Center, room 384-H (fourth floor)
Dinner available at 6:15pm

Algebraic Varieties: Connecting Algebra and Geometry

Aaron Kaufer

Mentor: Dan Dore

Geometric curves and shapes have natural deep connections to algebraic equations. For example, the unit circle is given by the two-variable equation $x^2 + y^2 = 1$. In this talk, we generalize this example by looking at sets of points defined as solution sets to systems of polynomial equations, known as affine algebraic varieties. In addition, we look at how studying the algebraic structure of the polynomial rings that these varieties come from can give insight into the geometric picture – one of the key insights of algebraic geometry.

The Calderón-Zygmund decomposition

Huy Pham

Mentor: Kevin Yang

The Calderón Zygmund decomposition is a useful tool in analysis, which can be used to give estimates on singular integral operators. In this presentation, I will apply the decomposition to give a lower bound on the probability that a multidimensional Brownian motion hits a set of positive measure before leaving a bounding box. If time permits, I will discuss some other applications as well.

Curvature, Genus, and the Gauss-Bonnet Theorem

Matthew Stevens

Mentor: Jesse Madnick

We begin by introducing notions of curvature for curves and Gaussian curvature for surfaces (as well as a geometric way of calculating Gaussian curvature). We continue by introducing the concept of triangulation and the Euler characteristic for surfaces, along with a theorem relating the Euler characteristic of a closed, connected, orientable surface to its genus. Finally, we present the Gauss-Bonnet theorem for closed surfaces, which, informally, allows us to conclude that even though the Gaussian curvature at a point on a closed, connected, orientable surface will likely change if we "bend" the surface, the integral of Gaussian curvature won't change and can be determined entirely by genus.

The Arnold Conjectures

Luis Kumanduri

Mentor: Abigail Ward

In symplectic geometry, there is a special class of diffeomorphisms called Hamiltonian diffeomorphisms which admit surprising structure. Some of this structure is captured by a collection of conjectures known as the Arnold conjectures. In my talk I'll introduce the fundamental objects at play in this story and give some examples to motivate these conjectures and why we might expect them to be true. Time permitting, I'll sketch some ideas behind the construction of Hamiltonian Floer homology and discuss how it can be use to establish the weak Arnold conjecture in special cases.

Doing mathematics inside a topos

Ben Heller

Mentor: Joj Helfer

In mathematical logic, (elementary) topoi are interesting to study since each topos gives us a new way to interpret logical formulas. A topos is a certain kind of category that generalizes the category of sets. I will define the topos and the internal logic of a topos, and explain the relationship with constructivism. Time permitting, I will define the natural number object of a topos and briefly describe a particular instance of doing mathematics inside a topos: how the effective topos gives rise to computable analysis (and the "theorem" that all functions $\mathbb{R} \to \mathbb{R}$ are continuous).

This event was financially supported by the Vice Provost for Graduate Education through the Student Projects for Intellectual Community Enhancement (SPICE) and Diversity and Inclusion Innovation Funds (DIF) programs. The organizers would like to thank Gretchen Lantz, Rose Stauder, and Elizabeth Kay for their administrative support of the DRP.