Stanford Math Directed Reading Program Colloquium

Winter 2017, Session 2

Thursday, April 6, 2017, 6:00pm–8:00pm
Sloan Mathematics Center, rooms 384-I and 384-H (fourth floor)
Dinner available at 5:45pm

Session A, room 384-I

Genus 0 and 1

6:00pm

Nitya Mani

Mentor: Jesse Silliman

There are many deep connections between algebra/arithmetic, geometry, and topology, and one of which is the curve/topological space invariant known as genus. I will give surprisingly different (but equivalent) characterizations of genus 0 and 1 objects from geometric and arithmetic perspectives, and along the way, highlight some of the major results that characterize genus 0 and 1 curves.

The classification theorem for compact surfaces

6:20pm

Nithin Kannan

Mentor: Cedric De Groote

In this talk, I will be presenting the Classification Theorem for all compact surfaces. We will start by looking at Euler characteristic and orientability: we will show that these are not only invariants under homeomorphism, but also the only invariants we need to classify any compact surface. Once we have done this, we will use the fundamental polygon of any given surface to show that it is homeomorphic to either (1) a sphere, (2) a connected sum of tori, or (3) a connected sum of real projective planes.

A possible one-way function from Chebyshev polynomials

6:40pm

Patrick DeMichele

Mentor: David Sherman

Essential to cryptography is the notion that some functions are "easy" to compute yet "hard" to invert. I will introduce this concept of "one-way functions" from a computability perspective, and demonstrate some cryptographic examples and applications. Finally, I will give a walk-through of a protocol using Chebyshev polynomials which is likely a one-way function.

Sharp thresholds for connectivity in Erdős–Rényi random graphs

7:00pm

Madelyne Xiao
Mentor: Yuval Wigderson

The Erdős–Rényi model is a paradigm for studying random graphs that defines a graph given some probability p that each edge in this graph exists.

Imagine that we can adjust p on a sliding scale (from 0 to 1) and observe the graphs that result for each possible value for p; a classic result of Erdős–Rényi says that, above a certain value for p, our random graph will be connected with probability 1 and, below such a threshold, our random graph will be connected with probability 0. That is to say, p is a sharp threshold for connectivity in random graphs. I will present the proof for this result and discuss sharp thresholds more broadly, time allowing!

The duality principle

7:20pm

Zoe Himwich
Mentor: Joj Helfer

Category theory is a subject that allows us to discuss a variety of structures in generalized terms. There are many concepts in category theory that can be considered "dual" to one another. This presentation will discuss many examples of duality in category theory, including monomorphisms and epimorphisms, covariant and contravariant functors, initial and terminal objects, and the use of duality to shorten a proof.

Neural networks on psychedelics

7:40pm

Nicolas Nuñez
Mentor: Brian Lawrence

Psychedelic drugs have the potential of bridging the gap between our neurochemistry and our subjective experiences. While research on psychedelic drugs usually relies on clinical trials and studies on rodent subjects, recently there has been an interest in studying the differences in subjective, written accounts of psychedelic experiences to understand similarities across different hallucinogenic drugs. In this presentation, I hope to describe how a Feed Forward Back Propagation (FFBP) Neural Network can be used to match written reports of psychedelic experiences with the hallucinogen that induced the experience. I will briefly explain classification problems in a general sense, and also in the context of FFBP neural networks.

Session B, room 384-H

Category theory and universal properties

6:00pm

Brian Morris

Mentor: Tony Feng

Category theory gives us a very abstract, general outlook for thinking about mathematical objects, as well as a language for extremely general statements and constructions. In this talk, we'll look at some practical (well, "practical") things that we can do with that outlook. We'll focus in particular on the universal property of the product, how we get from the familiar Cartesian product of sets to its generalization in category theory, and why this abstraction is appealing and useful.

Lattice cryptography and one-way hash functions

6:20pm

Katy Woo

Mentor: Jonathan Love

Lattice cryptography is an emerging field in cryptography which has gained attention for remaining secure against all known quantum computing attacks. I will present an introduction to some hard problems in lattices and their applications, specifically the one way hash function based on Ajtai's Construction.

An introduction to Riemannian geometry: exponential maps and the Hopf–Rinow theorem

6:40pm

Jaydeep Singh

Mentor: Evangelie Zachos

In this talk we will introduce some of Riemannian geometry's foundational concepts and constructions. In particular, we will study geodesics and the exponential map in order to better understand the local and global properties of Riemannian manifolds. Motivated by examples of non-Euclidean geometries, we will finally frame the famous local/global result, the Hopf–Rinow theorem, and discuss its ramifications in Riemannian geometry.

Phase portrait analysis of the Fitzhugh–Nagumo model

7:00pm

Bora Uyumazturk
Mentor: Alex Dunlap

One of the earliest milestones in the field of computational neuroscience was the introduction in 1961 of the Fitzhugh–Nagumo model, a system of planar differential equations which isolates the phenomenon of "excitability" of neurons observed in the more complicated Hodgkin-Huxley equations. My presentation will apply various techniques from nonlinear dynamics to the Fitzhugh–Nagumo system, culminating in a discussion of limit cycles as well their connection to the Poincare–Bendixson Theorem, a central result in the study of nonlinear planar systems.

A brief introduction to random matrix theory

7:20pm

Zhaolin Ren
Mentor: Erik Bates

The study of random matrices is an increasingly active research area, and has important applications in fields as diverse as physics, number theory, computer science, and statistics. In this talk, I will introduce the notion of random matrices by focusing on the Wigner random matrices, an important ensemble in random matrix theory. I will also discuss Wigner's semicircular law, a celebrated result in random matrix theory, and sketch out a proof using the moment method. Finally, if time permits, I hope to talk about how random matrices relate to the field of compressive sensing, itself an extremely active research field that has proved useful in a whole host of practical applications, most notably in magnetic resonance imaging.

Index theorems in topology

7:40pm

Luis Kumanduri
Mentor: Abigail Ward

The Atiyah–Singer Index Theorem is a deep and important theorem linking analytic data (the index of an elliptic operator) with topological data. In this talk we'll discuss elliptic operators and some of their basic properties, and I'll sketch a proof of the Toeplitz Index Theorem which is an important special case of the Atiyah–Singer theorem.

The organizers would like to thank Professor Brian Conrad, Gretchen Lantz, and Samantha Stone of the Stanford Mathematics Department for their hard work on behalf of the DRP, without which the program would not have been possible. This event was financially supported by the Mathematics Department and by the Vice Provost for Graduate Education through the Diversity and Inclusion Innovation Funds (DIF) program.