# Stanford Math Directed Reading Program Colloquium
## Winter 2017, Session 1

March 16, 2017, 6:00pm–8:00pm
Sloan Mathematics Center, Room 384-I (fourth floor)
Dinner available at 5:45pm

### Finite group representations and Schur's lemma — 6:00pm

Jack Lindsey

Mentor: Jens Reinhold

Understanding groups is, in general, a hard problem. We would benefit from a way to translate questions about groups to questions about a better understood class of objects: vector spaces. I will give a sense of how this is done by introducing finite group representations and building up to Schur's Lemma, in some sense the first nontrivial result of the theory. Throughout, I will connect the discussion to the example of low-order symmetric groups to illustrate some simple applications of representation theory.

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### Adventures in Teichmüller space — 6:20pm

Jared Bitz

Mentor: Weston Ungemach

A major front of research in modern hyperbolic geometry is the study of Teichmüller space, which is roughly the space of all hyperbolic structures with which we can imbue a given surface. In this talk, we'll gain some intuition for this area by studying a similar and more approachable question about the space of flat (e.g. Euclidean) structures on the torus.

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### Using degree theory to solve classic topology problems — 6:40pm

Pengda Liu

Mentor: Daniel Álvarez-Gavela

The Hairy Ball Theorem and Brouwer's fixed point theorem are both very interesting theorems in topology. They can be proved individually with some specific techniques. However, these theorems can easily be proved together by the theory of degree with no special techniques, but just some basic properties about degree. In this talk we will describe the notion of degree, its properties, and its applications.

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## Numbers in Arithmetic Progressions

7:00pm

Raj Raina

Mentor: Xiaoyu He

A classical question in Ramsey Theory is the following: is there such an $N$ so that given the integers in $[1, N]$, it is possible to color each number with one of $r$ colors such that there will always be a monochromatic $k$-term arithmetic progression? In 1927, Van Der Waerden showed that such an $N$ exists for any positive integers $r$ and $k$; in general, however, it is very hard to find the smallest such $N$ (even asymptotically) for values of $r$ and $k$.

In 1936, Erdos and Turan posed the stronger conjecture that for any integer $k$ and density $\delta > 0$, there is some $N$ so that any subset of $[1, N]$ with density at least $\delta$ contains a $k$-term arithmetic progression. This conjecture was proven by Szemeredi in 1975 and is considered a milestone in combinatorics.

In my talk, I will briefly go over the history of these kinds of problems and prove Behrend's Theorem, which is the construction of a surprisingly large set in $[1, N]$ without 3-term arithmetic progressions. The proof relies on the fact that a line cannot intersect a sphere in 3 places.

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## $(\infty, 1)$ **categories**

7:20pm

Scott Mutchnik

Mentor: Gergely Szucs

Higher category theory is a way of understanding higher homotopy theory by combining algebra and geometry. In this talk I will first motivate the underlying geometric structure for higher categories, the simplicial set. I will then explore two different approaches to defining higher categories, in particular $(\infty, 1)$ categories: structures with objects, arrows, invertible arrows between arrows, and so on (with higher arrows continuing to be invertible). The first approach views higher categories directly as simplicial sets, requiring enough homotopies for compositions to be defined and higher arrows to be invertible. The second approach views higher categories as analogous to ordinary categories, but where the arrows between any two objects form $\infty$-groupoids, a simpler notion, rather than sets. If time permits I will then relate the two notions via an adjunction.

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## About the DRP

The Directed Reading Program is a program of the Stanford Mathematics Department in which undergraduate students independently read some mathematics outside of their official coursework, mentored by math graduate students. For more information or to apply to participate in a future quarter, visit `http://mathdrp.stanford.edu`.

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